Kittel s State

6.3. The Fermi-Dirac distribution is given by

$$f(\varepsilon) = \frac{1}{\exp[(\varepsilon-\mu)/\tau]+1},$$

where μ is only dependent on $\tau = k_B T$, not on ε.

For conservation of fermion #, we impose

$$\int_0^\infty D(\varepsilon) f(\varepsilon)\, d\varepsilon = n = \frac{N}{L^2},$$

Then $$\int_0^\infty \frac{m}{\pi\hbar^2} f(\varepsilon)\, d\varepsilon = n,$$

$$\int_0^\infty f(\varepsilon)\, d\varepsilon = \frac{n\pi\hbar^2}{m}.$$

Observe that $$f(\varepsilon) = \frac{1}{\exp[(\varepsilon-\mu)/\tau]+1} = \frac{\exp[(\mu-\varepsilon)/\tau]}{\exp[(\mu-\varepsilon)/\tau]+1},$$

It's then clear that $$f(\varepsilon) = \frac{\partial}{\partial\varepsilon}\left[\tau \ln\left\{\exp[(\mu-\varepsilon)/\tau]+1\right\}\right].$$

$$\Rightarrow \int_0^\infty f(\varepsilon)\, d\varepsilon = -\tau\ln\left\{\exp[(\mu-\varepsilon)/\tau]+1\right\}\Big|_0^\infty$$

$$= \tau\ln\left\{\exp[(\mu-\varepsilon)/\tau]+1\right\}\Big|_0$$

$$= \tau\ln\left\{\exp[\mu/\tau]+1\right\}$$

$$\Rightarrow \tau\ln\left\{\exp[\mu/\tau]+1\right\} = \frac{n\pi\hbar^2}{m},$$

$$\boxed{\mu = \tau\ln\left\{\exp\left[\frac{n\pi\hbar^2}{m\tau}\right]-1\right\}}$$

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